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M.A./M.Sc. FIRST SEMESTER EXAMINATION, MARCH 2022
FIRST YEAR [BATCH 2021-23]
Mathematics

Date : 25/03/2022 **Topology**
Time : 11.00 am – 1.00 pm **Paper** : MTM CC4 Full Marks:50

Answer all the questions, maximum one can score 50.

1. (a) Show that in the Sorgenfrey line, $(0,\infty)$ is an open set. [2]
   (b) Let $\tau = \{U \subseteq \mathbb{R} :\ 0 \notin U \text{ or } \mathbb{R} - U \text{ is finite}\}$. Show that τ is a topology on $\mathbb{R}$. Also find cl $\mathbb{N}$ in $(\mathbb{R}, \tau)$. [2+2]
   (c) Suppose τ and τ_l respectively denote the usual topology and the lower limit topology on $\mathbb{R}$. Show that a subset D of $\mathbb{R}$ is dense in $(\mathbb{R}, \tau)$ iff it is dense in $(\mathbb{R}, \tau_l)$. [3]
   (d) Give examples of two closed sets A and B in $\mathbb{R}$ with usual topology such that $A + B = \{x + y :\ x \in A,\ y \in B\}$ is not closed in $\mathbb{R}$. Justify your answer. [3]

2. (a) Consider the topology $(\mathbb{R}, \tau)$ where $\tau = \{U \subseteq \mathbb{R} :\ \mathbb{Q} \subseteq U\} \cup \{\emptyset\}$. Verify whether $(\mathbb{R}, \tau)$ is 1st countable, separable and Lindelöf. [3+2+2]
   (b) Is the space $(\mathbb{R}, \tau)$ of 2(a) metriziable? Justify your answer. [2]

3. (a) Let $\mathbb{R}^* = \mathbb{R} - \{0\}$. Suppose τ_c denotes the cocountable topology on $\mathbb{R}^*$ and τ denotes the usual topology on $\mathbb{R}$. Let $f : (\mathbb{R}^*, \tau_c) \to (\mathbb{R}, \tau)$ be defined by $f(x) = \frac{1}{x}\ \forall x \in \mathbb{R}^*$. Verify whether f is open, closed and continuous. [2+2+3]
   (b) Suppose $f, g : \mathbb{R} \to \mathbb{R}$ be two continuous maps such that $f(x) = g(x)\ \forall x \in \mathbb{Q} - \mathbb{N}$. Prove that $f = g$. [3]
   (c) Let $f : \mathbb{R} \to \mathbb{R}$ be continuous and $A = \{x \in \mathbb{R} : f(x) \geq 1\}$. Show that $\exists$ a continuous map $h : \mathbb{R} \to \mathbb{R}$ such that $A = \{x \in \mathbb{R} : h(x) = 0\}$. [4]

4. (a) Show by examples that normality is neither expansive nor contractive. [4]
   (b) Prove that normality is a topological property. [3]

5. (a) Suppose τ_f and τ respectively denote the cofinite topology and the usual topology on $\mathbb{R}$. Show that there is no continuous bijection $g : (\mathbb{R}, \tau_f) \to (\mathbb{R}, \tau)$. [4]
   (b) Is $\mathbb{N}$ is a compact set in $\mathbb{R}$ with cocountable topology? Justify your answer. [3]

6. (a) Find all components of (X, τ) where $X = \left\{0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\right\}$ and τ is the usual topology. [3]
   (b) Prove that there is no metric 'd' on $\mathbb{N}$ such that $(\mathbb{N}, d)$ is connected. [4]

7. Find the cardinal number of $\mathscr{A}$ where $\mathscr{A} = \{D \subseteq \mathbb{Q} : D \text{ is dense in } \mathbb{R}\}$. [4]

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